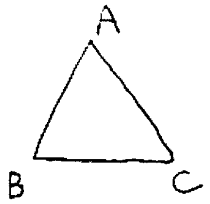
Windomの解答速報 日本医科大学 数学

2015 日医

$\boxed{1}$

問1.



(1) n秒後に $\qquad\qquad$ $(n+1)$秒後

$\begin{cases} \text{Aにいる } P_n \xrightarrow{\frac{1}{4}} \text{Aにいる } P_{n+1} \\ \text{Aにいない } 1-P_n \xrightarrow{\frac{3}{8}} \end{cases}$

$$P_{n+1}=\frac{1}{4}P_n+\frac{3}{8}(1-P_n) \qquad \therefore \underline{P_{n+1}=-\frac{1}{8}P_n+\frac{3}{8}}$$

(2) (1)より

$$P_{n+1}-\frac{1}{3}=\left(P_1-\frac{1}{3}\right)\cdot\left(-\frac{1}{8}\right)^{n-1}$$

$$\therefore \underline{P_n=\frac{2}{3}\cdot\left(-\frac{1}{8}\right)^n+\frac{1}{3}}$$

(3) $\displaystyle\lim_{n\to\infty}P_n=\underline{\frac{1}{3}}=P$

(4) $|P_n-P|<5^{-20}$ より、 $\dfrac{2}{3}\cdot\left(\dfrac{1}{8}\right)^n<\left(\dfrac{1}{5}\right)^{20}$

対数(底10)をとると

$$\log_{10}2-\log_{10}3-3n\log_{10}2<-20(1-\log_{10}2)$$

$$\therefore n>15.29\cdots$$

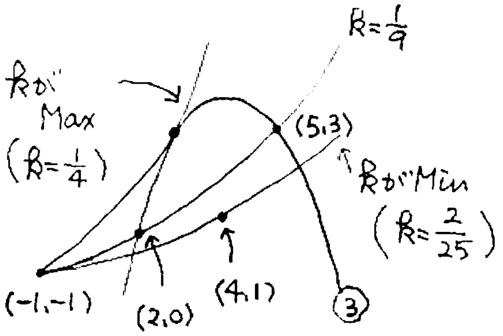
$$\therefore \underline{n=16}$$

問2

点(x, y)

$y \geqq \frac{1}{2}x-1 \cdots ①$、$y \geqq 2x-7 \cdots ②$、$y \leqq -x^2+8x-12 \cdots ③$

(1) $\frac{y+1}{(x+1)^2} = k \iff y+1 = k(x+1)^2$ とおくと



kがMax … ③と接するときで、$k=\frac{1}{4}$　　kがMin … $\underline{(4,1)}$を通るときで、$\underline{k=\frac{2}{25}}$

このとき

$\underline{(x, y) = (3, 3)}$

(2) $\frac{y+1}{(x+1)^2} + \frac{(x+1)^2}{y+1} = k + \frac{1}{k} \ (= f(k))$　$\left(\begin{array}{l} \text{ただし} \\ \frac{2}{25} \leqq k \leqq \frac{1}{4} \end{array}\right)$

$f'(k) = 1 - \frac{1}{k^2} = \frac{(k-1)(k+1)}{k^2}$

x	$\frac{2}{25}$		$\frac{1}{4}$
f'		$-$	
f	$\frac{629}{50}$	↘	$\frac{17}{4}$

よって、$\underline{\text{Max}\ \frac{629}{50},\ \text{Min}\ \frac{17}{4}}$

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問1. $\displaystyle \lim_{n\to\infty}\left(\frac{1}{n+\frac{1}{2}}+\frac{1}{n+\frac{2}{2}}+\cdots+\frac{1}{n+\frac{2n}{2}}\right)=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{2n}\frac{1}{1+\frac{k}{2n}}$

$$=\int_0^2\frac{1}{1+\frac{x}{2}}dx=\Big[2\log(2+x)\Big]_0^2=\underline{2\log 2}$$

問2. $\displaystyle \lim_{n\to\infty}\left(\frac{1}{n+\frac{1}{2}}+\frac{1}{n+\frac{3}{2}}+\cdots+\frac{1}{n+\frac{4n-1}{2}}\right)=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{2n}\frac{1}{1+\frac{2k-1}{2n}}$

ここに

$$\frac{1}{1+\frac{2k}{2n}}<\frac{1}{1+\frac{2k-1}{2n}}<\frac{1}{1+\frac{2(k-1)}{2n}}$$

$$\sum_{k=1}^{2n}\frac{1}{1+\frac{2k}{2n}}<\sum_{k=1}^{2n}\frac{1}{1+\frac{2k-1}{2n}}<\sum_{k=1}^{2n}\frac{1}{1+\frac{2(k-1)}{2n}}$$

$\therefore\ \displaystyle\int_0^2\frac{1}{1+x}dx<$ 与式 $<\displaystyle\int_0^2\frac{1}{1+x}dx$

よって　与式 $=\underline{\log 3}$

問3　与式の対数をとると

$$\lim_{n\to\infty}\frac{1}{n}\left\{\log\left(1+\sin\frac{n\pi}{2n}\right)^{\sin\frac{n\pi}{n}}+\cdots+\log(1+\sin\pi)^{\sin 2\pi}\right\}$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n}\log\left(1+\sin\frac{\pi}{2}\left(1+\frac{k}{n}\right)\right)^{\sin\left(1+\frac{k}{n}\pi\right)}$$

$$=\int_0^1\log\left(1+\sin\frac{\pi}{2}(1+x)\right)^{\sin(1+x)\pi}dx$$

$$=\int_0^1(-\sin\pi x)\log\left(1+\cos\frac{\pi}{2}x\right)dx$$

$$=\int_2^1\frac{4}{\pi}(t-1)\log t\,dt\qquad\left(1+\cos\frac{\pi}{2}x=t\text{ とおく}\right)$$

$$=-\frac{1}{\pi}$$

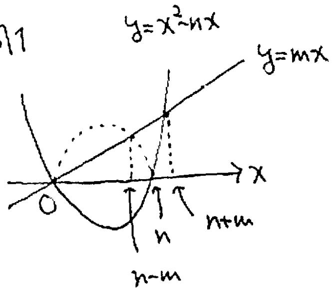
$$=\log e^{-\frac{1}{\pi}}$$

よって、与式 $=\underline{e^{-\frac{1}{\pi}}}$

$\boxed{3}$

$$\begin{cases} y = x^2 - nx \\ y = mx \\ n > 1,\ m > 0,\ n > m,\ n > \frac{1}{m} \end{cases}$$

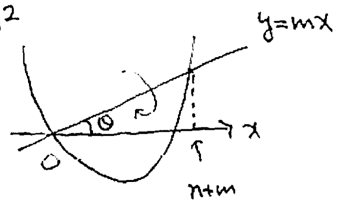
問1



$$V_n = \int_0^{n-m} \pi(-x^2+nx)^2 dx + \int_{n-m}^{n+m} \pi(mx)^2 dx - \int_n^{n+m} \pi(x^2-nx)^2 dx$$

$$= \underline{\pi\left(\frac{1}{30}n^5 + \frac{4}{15}m^5 + \frac{4}{3}m^3n^2\right)}$$

問2



$$W_n = \pi \underbrace{\cos\theta}_{\downarrow\ \frac{1}{\sqrt{m^2+1}}} \int_0^{n+m} \{(x^2-nx)-mx\}^2 dx$$

$$= \frac{\pi}{30}\cos\theta\,(n+m)^5$$

$$= \underline{\frac{\pi(n+m)^5}{30\sqrt{m^2+1}}}$$

問3

$$\lim_{n\to\infty} \frac{V_n}{W_n} = \frac{\frac{\pi}{30}}{\frac{\pi}{30}\cdot\frac{1}{\sqrt{m^2+1}}} = \underline{\sqrt{m^2+1}}$$