1

$\cos(\alpha+\beta)=\cos\alpha-1$ ・・・・・・・・・・・・・①

$\cos(\alpha+\beta)=\cos\beta-1$ ・・・・・・・・・・・・・②

①，②より

$\cos\alpha-1=\cos\beta-1$

$\cos\alpha=\cos\beta$

$0\leq\alpha,\ \beta\leq\pi$ より　$\alpha=\beta$

①（又は②）は　$\cos 2\alpha=\cos\alpha-1$

$\Leftrightarrow\quad 2\cos^2\alpha-1=\cos\alpha-1$

$2\cos^2\alpha-\cos\alpha=0$

$(2\cos\alpha-1)\cos\alpha=0$

$\cos\alpha=\dfrac{1}{2},\quad 1$

$\alpha=\dfrac{\pi}{3},\quad \dfrac{\pi}{2}$

$(\alpha,\ \beta)=\boxed{\left(\dfrac{\pi}{3},\ \dfrac{\pi}{3}\right),\ \left(\dfrac{\pi}{2},\ \dfrac{\pi}{2}\right)}$

(1)

2

（ⅰ）　$y=x^2-2x-3=(x-1)^2-4$　　頂点 $(1,\ -4)$

$y=x^2$ の焦点は　$\left(0,\ \dfrac{1}{4}\right)$

なので，

$y=x^2-2x-3$ の焦点は

$\left(0+1,\ \dfrac{1}{4}-4\right)=\boxed{\left(1,\ -\dfrac{15}{4}\right)}$

(2)

⊛焦点が $(0,\ p)$

準線が $y=-p$

$\Rightarrow\quad 4py=x^2$

（ⅱ） $y = x^2 - 2x - 3$

$\Downarrow$ y 軸対称

$y = x^2 + 2x - 3$

$\Downarrow$ $y = x$ に関して対称

$\boxed{x = y^2 + 2y - 3}$

(3)

$\left(1,\ -\dfrac{15}{4}\right)$

$\Downarrow$ y 軸対称

$\left(-1,\ -\dfrac{15}{4}\right)$

$\Downarrow$ $y = x$ に関して対称

$\boxed{\left(-\dfrac{15}{4},\ -1\right)}$

(4)

3

（ⅰ） $W(x) = \int_0^x e^{-a(x-s)} f(s)ds = \int_0^x e^{-ax} \cdot e^{as} f(s)ds$

$= e^{-ax} \int_0^x e^{as} f(s)ds$

$W'(x) = -ae^{-ax} \int_0^x e^{as} f(s)ds + e^{-ax} \times e^{ax} f(x)$

$= -a\underline{\int_0^x e^{-a(x-s)} f(s)ds} + f(x)$

$\parallel$

$W(x)$より

$W'(x) = -aW(x) + f(x)$

$\Leftrightarrow$ $\boxed{W'(x) + aW(x)}$ $= f(x)$ ・・・・・・・・・・・・・①

(5)

（ⅱ） $W'(x) + W(x) = x^2$

①と比較して $a = 1$， $f(x) = x^2$ とする

$W = \int_0^x e^{-(x-s)} f(s)ds$

$= e^{-x} \int_0^x e^s s^2 ds$

$= e^{-x} \left[e^s (s^2 - 2s + 2)\right]_0^x$

(6)

$= e^{-x} \left\{e^x (x^2 - 2x + 2) - 1 \times 2\right\} = \boxed{(x^2 - 2x + 2) - 2e^{-x}}$

(ⅲ) ① $\Leftrightarrow$ $W'(x)=f(x)-aW(x)$ ・・・・・・・・・・・・・②

xで微分すると

$W''(x)=f'(x)-aW'(x)$

②と $f'(x)=g(x)$ を代入すると

$W''(x)=g(x)-a\bigl(f(x)-aW(x)\bigr)$

$\boxed{=a^2W(x)-af(x)+g(x)}$

(7)

$\boxed{4}$

$$m\frac{d^2x_1}{dt^2}=-k(x_2-x_1)+kl \quad \cdots\cdots ①$$

$$m\frac{d^2x_2}{dt^2}=k(x_2-x_1)-kl+F \quad \cdots\cdots ②$$

(ⅰ) $x_2-x_1=u$ とおく

$$\frac{du}{dt}=\frac{dx_2}{dt}-\frac{dx_1}{dt},\quad \frac{d^2u}{dt^2}=\frac{d^2x_2}{dt^2}-\frac{d^2x_1}{dt^2} \quad \cdots\cdots ③$$

②−①より

$$m\left(\frac{d^2x_2}{dt^2}-\frac{d^2x_1}{dt^2}\right)=2k(x_2-x_1)-2kl+F$$

$$\overset{③}{\Leftrightarrow} \quad m\frac{d^2u}{dt^2}=\boxed{2ku-2kl+F}$$

(8)

(ⅱ) $x_2+x_1=v$ とおく

$$\frac{dv}{dt}=\frac{dx_2}{dt}+\frac{dx_1}{dt} \qquad \frac{d^2v}{dt^2}=\frac{d^2x_2}{dt^2}+\frac{d^2x_1}{dt^2} \quad \cdots\cdots ④$$

②+①より

$$m\left(\frac{d^2x_2}{dt^2}+\frac{d^2x_1}{dt^2}\right)=F$$

$$\overset{④}{\Leftrightarrow} \quad m\frac{d^2u}{dt^2}=\boxed{F}$$

(9)

(ⅲ)

$$\boxed{\begin{array}{l} W''(x)=a^2W(x)-af(x)+f'(x) \quad \cdots\cdots ⑤ \\ \qquad \Rightarrow W(x)=\int_0^x e^{-a(x-s)}f(s)ds \end{array}}$$

(道具)

$\boxed{u\text{について}}$

$$m\frac{d^2u}{dt^2} = 2ku - 2kl + F = 2ku - 2kl + c$$

$$u''(t) = \frac{2k}{m}u + \frac{c-2kl}{m}$$

$a^2 = \frac{2k}{m}$ とおく　　$a = \pm\sqrt{\frac{2k}{m}}$

$$-af(x) + f'(x) = \frac{c-2kl}{m}$$

$$-ae^{-ax}f(x) + e^{-ax}f'(x) = \frac{c-2kl}{m}e^{-ax}$$

x で積分

$$e^{-ax}f(x) = -\frac{c-2kl}{ma}e^{-ax} + E$$

（E :積分定数）

$$f(x) = -\frac{c-2kl}{ma} + Ee^{ax}$$

$$u = \int_0^t e^{-a(t-s)}f(s)ds$$

$$= \int_0^t e^{-a(t-s)} \times \left\{-\frac{c-2kl}{ma} + Ee^{as}\right\}ds$$

$$= e^{-at}\int_0^t e^{as}\left(-\frac{c-2kl}{ma} + Ee^{as}\right)ds$$

$$= e^{-at}\int_0^t \left(-\frac{c-2kl}{ma}e^{as} + Ee^{2as}\right)ds$$

$$= e^{-at}\left[-\frac{c-2kl}{ma^2}e^{as} + \frac{E}{2a}e^{2as}\right]_0^t$$

$$= e^{-at}\left\{\left(-\frac{c-2kl}{ma^2}e^{at} + \frac{E}{2a}e^{2at}\right) - \left(-\frac{c-2kl}{ma^2} + \frac{E}{2a}\right)\right\}$$

$$= -\frac{c-2kl}{ma^2} + \frac{c-2kl}{ma^2}e^{-at} + \frac{E}{2a}\left(e^{at} - e^{-at}\right)$$

u が発散しないので，　$E = 0$,　$a > 0$ より　$a = \sqrt{\frac{2k}{m}}$

$$u(t) = -\frac{c-2kl}{m \times \frac{2k}{m}} + \frac{c-2kl}{m \times \frac{2k}{m}}e^{-\sqrt{\frac{2k}{m}}t}$$

$$= \boxed{-\frac{c-2kl}{2k} + \frac{c-2kl}{2k} e^{-\sqrt{\frac{2k}{m}}t}}$$

(10)

$\boxed{v(x) \text{ について}}$

$$m\frac{d^2v}{dt^2} = F = c \quad \Leftrightarrow \quad V''(x) = \frac{c}{m}$$

⑤において　$a = 0$

$$-af(x) + f'(x) = \frac{c}{m}$$

$$f'(x) = \frac{c}{m} \qquad f(x) = \frac{c}{m}x + D \quad (D:\text{積分定数})$$

$$v(t) = \int_0^t e^{-0(t-s)} f(s)ds = \int_0^t f(s)ds$$

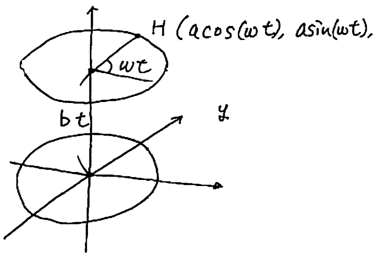
$$= \int_0^t \left(\frac{c}{m}s + D\right)ds$$

$$= \left[\frac{c}{2m}s^2 + Ds\right]_0^t$$

$$= \boxed{\frac{c}{2m}t^2 + Dt} \quad (D:\text{積分定数})$$

(11)

$\boxed{5}$



$$x = x(t) = a\cos(wt)$$
$$y = t(t) = a\sin(wt)$$
$$z = z(t) = bt$$

（ｉ）　$\text{OH}^2 = a^2\cos^2(wt) + a^2\sin^2(wt) + b^2t^2$

$$= a^2 + b^2t^2$$

(12)

$$\therefore \boxed{\text{OH} = \sqrt{a^2 + b^2t^2}}$$

（ⅱ）　$\text{OH} = d$

$$\sqrt{a^2 + b^2t_d^2} = d$$

$$a^2 + b^2t_d^2 = d^2$$

$$t^2d = \frac{d^2 - a^2}{b^2}$$

$td \geq 0$ より　$td = \boxed{\dfrac{\sqrt{d^2 - a^2}}{b}}$
(13)

(iii) $l = \int_0^{td} \sqrt{x'(t)^2 + y'(x)^2 + z'(t)^2}\, dt$

$x'(t) = -aw\sin wt$
$t'(t) = aw\cos wt$
$z'(t) = b$

$$= \int_0^{td} \sqrt{a^2w^2\sin^2 wt + a^2w^2\cos^2 wt + b^2}\, dt$$
$$= \int_0^{td} \sqrt{a^2w^2 + b^2}\, dt$$
$$= \sqrt{a^2w^2 + b^2}\, [t]_0^{td}$$
$$= \sqrt{a^2w^2 + b^2}\, td$$
$$= \sqrt{a^2w^2 + b^2} \times \frac{\sqrt{d^2 - a^2}}{b}$$
$$= \boxed{\frac{\sqrt{(a^2w^2 + b^2)(d^2 - a^2)}}{b}}$$
(14)

(iv) $|\overrightarrow{\mathrm{PQ}}|^2 = \{a\cos(w(t+h)) - a\cos(wt)\}^2 + \{a\sin(w(t+h)) - a\sin(wt)\}^2$

$$= a^2\left\{(\cos(w(t+h)) - \cos(wt))^2 + (\sin(w(t+h)) - \sin(wt))^2\right\}$$

$|z(t+h) - z(t)| = |b(t+h) - bt| = |bh|$

$$\lim_{h\to 0}\frac{|z(t+h) - z(t)|^2}{|\overrightarrow{\mathrm{PQ}}|^2}$$
$$= \lim_{h\to 0}\frac{b^2h^2}{a^2\left\{(\cos w(t+h) - \cos wt)^2 + (\sin w(t+h) - \sin wt)^2\right\}}$$
$$= \lim_{h\to 0}\frac{b^2}{\left\{\left(\dfrac{\cos w(t+h) - \cos wt}{h}\right)^2 + \left(\dfrac{\sin w(t+h) - \sin wt}{h}\right)^2\right\}}$$
$$= \frac{b^2}{a^2\left\{(-w\sin wt)^2 + (w\cos wt)^2\right\}}$$
$$= \frac{b^2}{a^2w^2}$$

$$\therefore \lim_{h\to 0}\frac{|z(t+h) - z(t)|}{|\overrightarrow{\mathrm{PQ}}|} = \sqrt{\frac{b^2}{a^2w^2}} = \boxed{\frac{b}{aw}}$$
(15)